THE INTERPLAY BETWEEN GEOSTATISTICAL PRIOR INFORMATION AND MODELING ERROR IN SEISMIC DATA

Rasmus Bødker Madsen
Niels Bohr Institute
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UNIVERSITY OF COPENHAGEN

- MOTIVATION
- FORWARD MODEL
- PRIOR MODEL
- FORWARD MODELING ERROR
- INVERSION RESULTS
- SUMMARY

Motivation

- Linearised AVO inversion is popular
- Linear forward (g)

$$\mathbf{d} = g(\mathbf{m}) \rightarrow \mathbf{d} = \mathbf{G}\mathbf{m}$$

  - Computationally efficient
  - Analytical solutions to inverse problem
- Assumptions
  - Prior model is Gaussian
- Is the subsurface really Gaussian?
  - What kind of errors do we introduce in the forward?
    - Can they be quantified?
    - Can we account for these errors?

Forward model

The basics

- Seismic signal
  - Approximated by convolutional model

$$S(t) = W(t) * R(t)$$

W(t): Seismisk wavelet

R(t): Seismisk reflektion koefficienter (seismisk forward)

- Reflection coefficients can be calculated by Zoeppritz equations
- Or approximated
  - E.g Aki & Richards, Shuey
  - Linear: Buland & Omre[1]

1) Buland, Arild, and Henning Omre. "Bayesian linearized AVO inversion." Geophysics 68.1 (2003): 185-198.

Buland & Omre - Linear forward

$$\mathbf{d}_{\mathrm{AVO}} = \mathbf{W} \cdot \mathbf{R} + \mathbf{e} = \mathbf{W} \cdot \mathbf{G} \cdot \mathbf{m}' + \mathbf{e}$$

- **W:** Wavelet matrix
- **R:** Seismic reflection coefficients
- **G:** Linear forward
- **m:** Model parameters

- Requirements: log-Gaussian prior distribution

$$\mathbf{m}(t) = [\ln \alpha(t), \ln \beta(t), \ln \rho(t)]^T$$

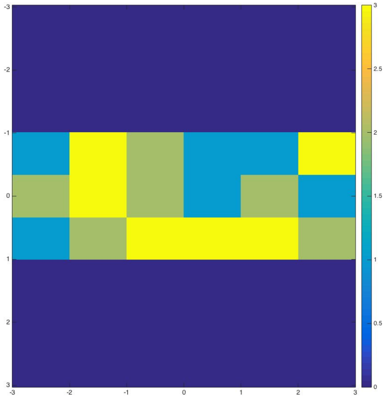
- Benefits: Linear inversion
- Problems: Introduce forward modeling error?

Prior model

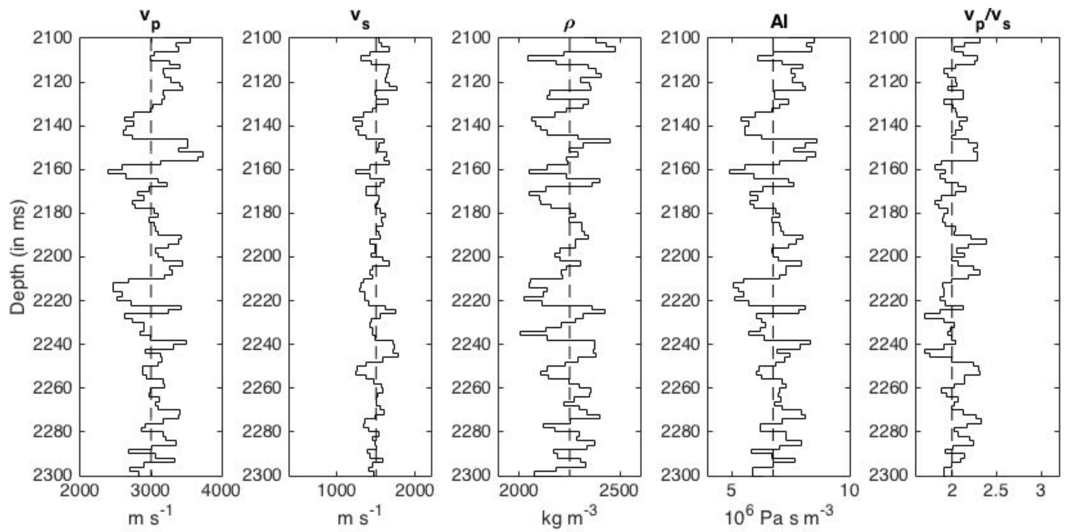
Prior model - Two cases

- ‘Best-case’
- Continuous Gaussian vector field
  - Methodology of Buland & Omre (2003)
  - Correlation between model parameters (cor = 0.7)
  - Spatial correlation (range = 5 ms)
  - ???? + 2 other covariance types
    - Cov ='1 Sph(1,0,5e-3)'
    - Cov ='1 Exp(1,0,5e-3)' ?????

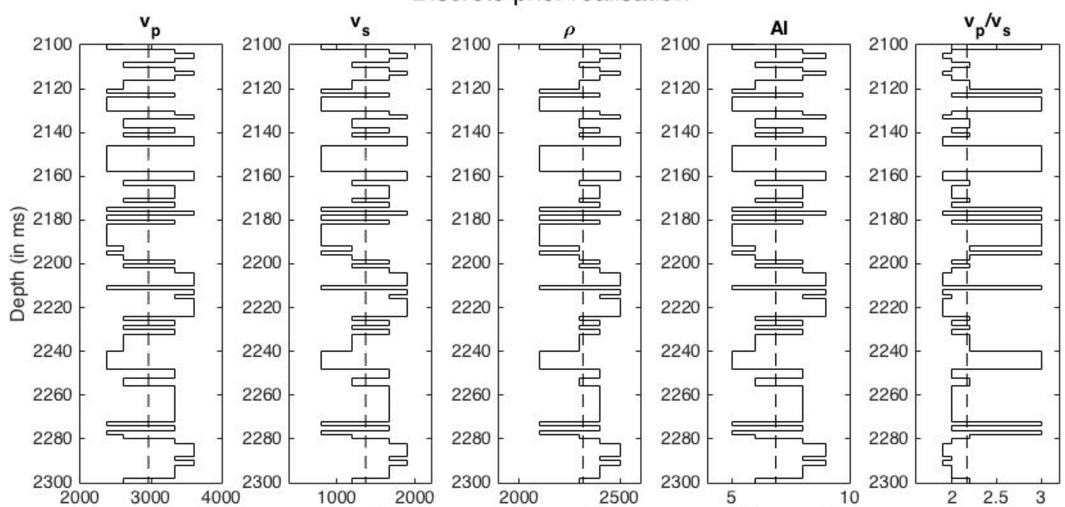
- ‘Worst-case’
- Discontinuous
  - Truncated pluri-Gaussian model
  - 4 layers
  - Covariance models
    - Cov ='1 Gau(1,0,5e-3)'
    - Cov ='1 Exp(10,0,5e-3)'



Reference model



Discrete prior realisation



Forward modeling error

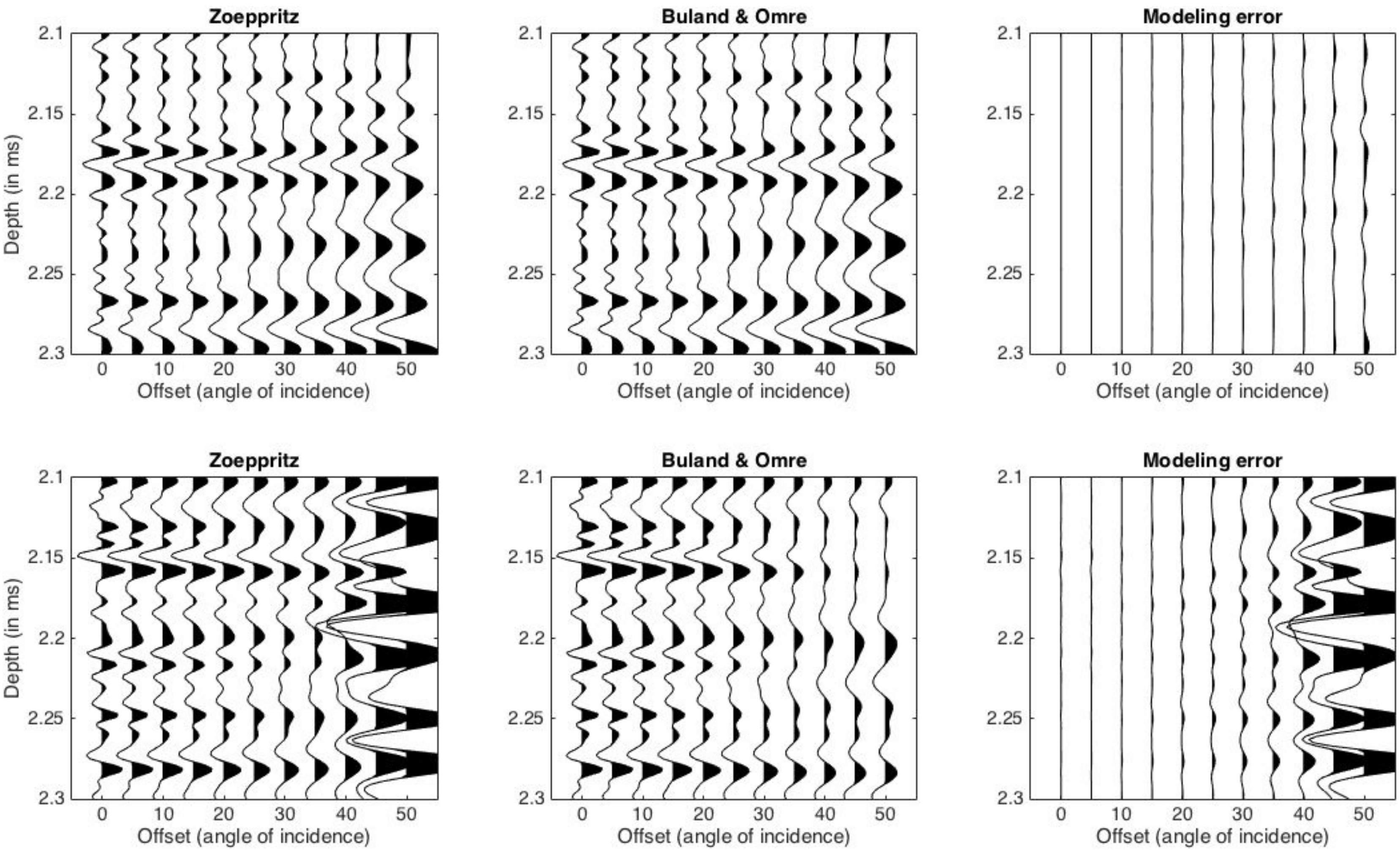
Forward modeling error

$$S_{error}(t) = S_{zoep}(t) - S_{app}(t)$$

$$S_{error}(t) = W * R_{zoep}(t) - W * R_{app}(t)$$

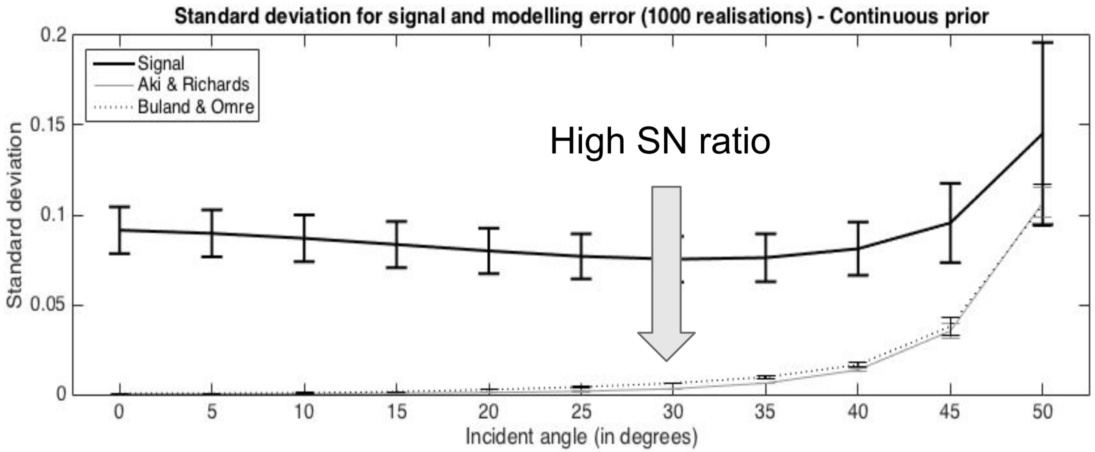
- Obtaining a sample of forward modeling error
  - A realisation of prior model
  - Calculate two forward responses: true and approx.
  - Convolve both seismic reflection series with wavelet
- A realisation of forward modeling error for a given prior model

One realisation

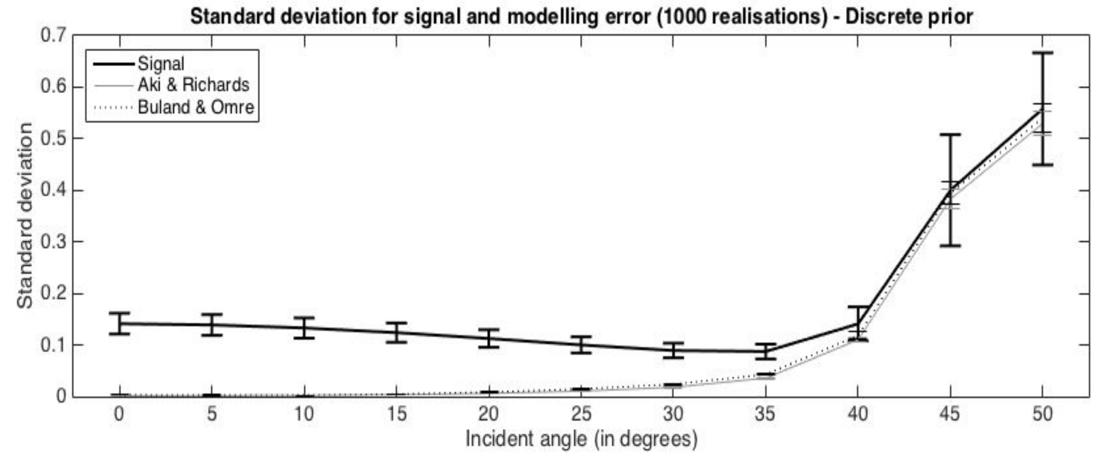


Forward modeling error sample

- 1000 realisations
  - Both prior models
- Standard deviation of modeling error sample at different incident angle



- Comparison
  - 1000 realisations of seismic AVO data calculated using zoeppritz equations as forward



Quantifying modeling error

Quantifying modeling error

- Adapt methodology proposed by Hansen et. al. (2014)[2]
- Estimate mean

$$\mathbf{d}_{T_{\text{app}}} = [\mathbf{d}^1_{T_{\text{app}}}, \mathbf{d}^2_{T_{\text{app}}}, \ldots, \mathbf{d}^N_{T_{\text{app}}}]$$

$$\text{where} \quad d^i_{T_{\text{app}}} = \frac{1}{N}\sum_{i=1}^{N}(D^{i,j}_{\text{ex}} - D^{i,j}_{\text{app}}),$$

- and covariance modeling error

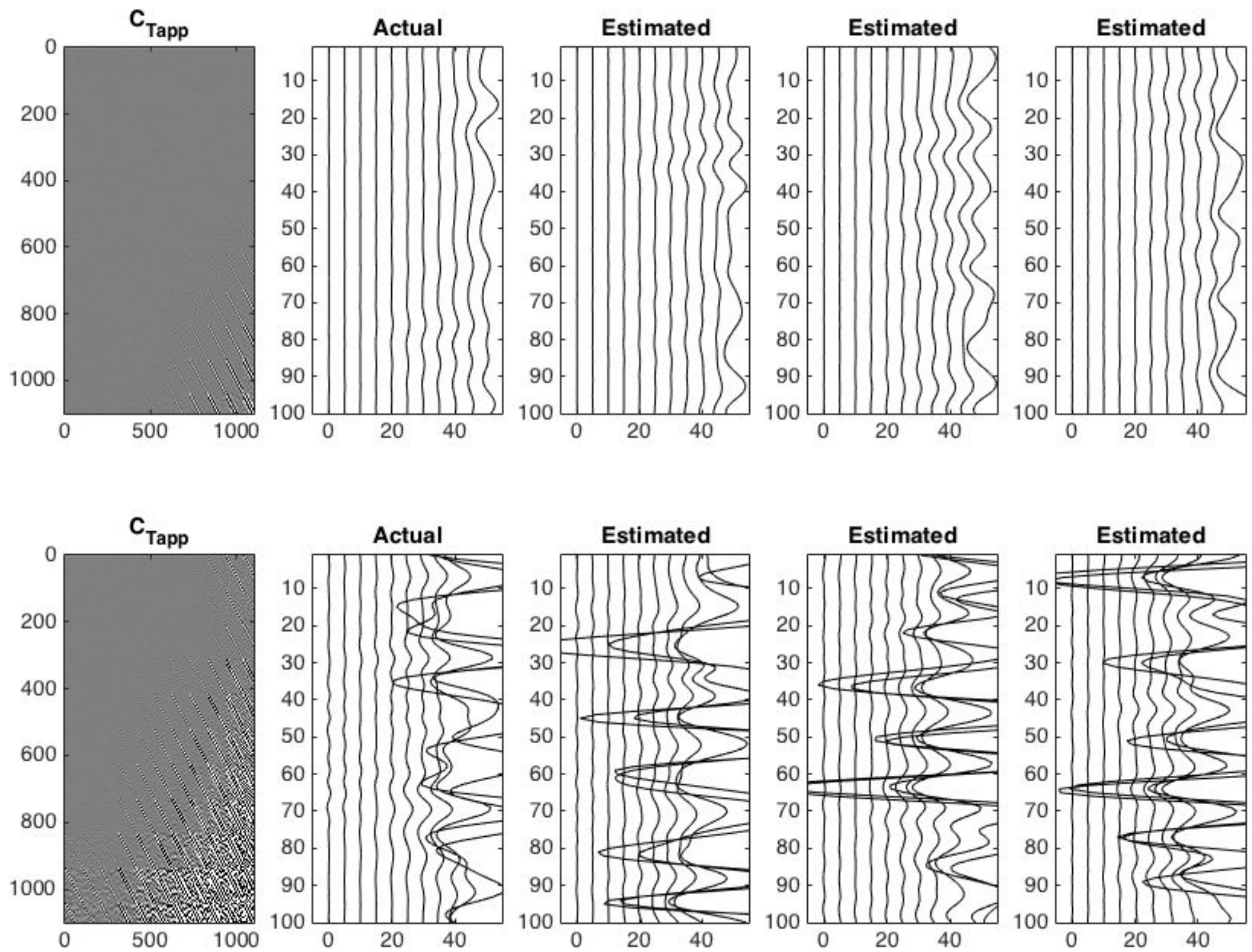
$$\mathbf{C}_{T_{\text{app}}} = \frac{1}{N}\mathbf{D}_{\text{diff}}\mathbf{D}'_{\text{diff}} \quad \text{where} \quad \mathbf{D}_{\text{diff}} = [\mathbf{D}_{\text{ex}} - \mathbf{D}_{\text{app}} - \mathbf{D}_{T_{\text{app}}}]$$

$$\text{and } \mathbf{D}_{T_{\text{app}}} = [\mathbf{d}'_{T_{\text{app}}}, \mathbf{d}'_{T_{\text{app}}}, \ldots, \mathbf{d}'_{T_{\text{app}}}].$$

2) Hansen, Thomas Mejer, et al. "Accounting for imperfect forward modeling in geophysical inverse problems—Exemplified for crosshole tomography." Geophysics 79.3 (2014): H1-H21.

Quantifying modeling error - Validation

- N = 1000 realisations



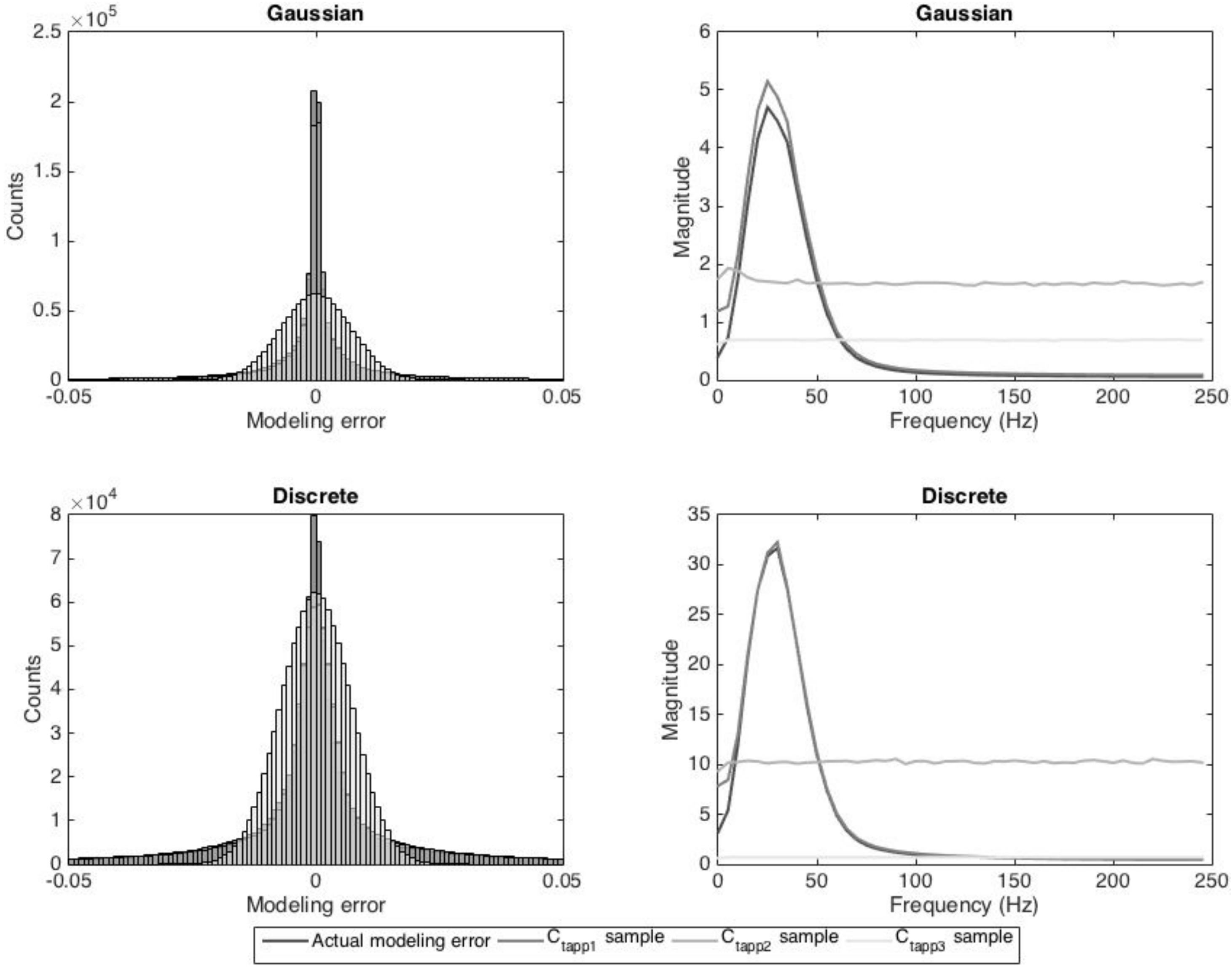
Quantifying modeling error - Covariance

- C_{Tapp1} : Estimated modeling error covariance
- C_{Tapp2} : Uncorrelated part (Diag(C_{Tapp1}))
- C_{Tapp3} : Constant variance ($\sigma^2 = 5e\text{-}5$)

- Sample each covariance matrix
  - Generate 1000 realisations

- Compare with actual modeling error
  - Histogram, frequency content, log likelihood distribution

Quantifying modeling error - Comparison

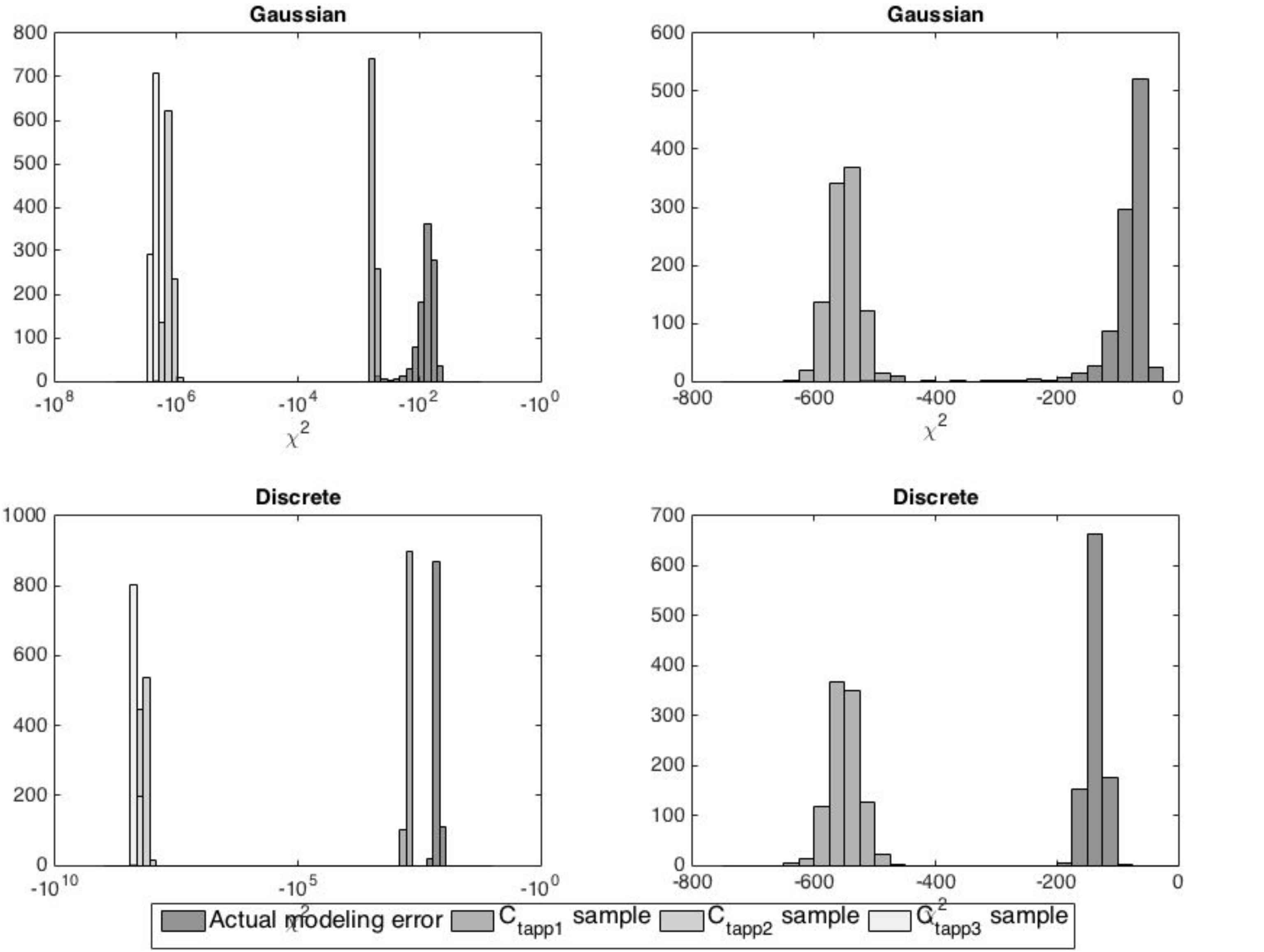


Quantifying modeling error - likelihood

- Calculating likelihood that realisations originate from the estimated modeling error covariance (C_{Tapp}) with the mean (d_{Tapp})

$$\log(L(\mathbf{m})) = -0.5\left(\mathbf{d}_{\text{test}} - \mathbf{d}_{\mathrm{T_{app}}}\right)^t \mathbf{C}_{\mathrm{T_{app}}}^{-1}\left(\mathbf{d}_{\text{test}} - \mathbf{d}_{\mathrm{T_{app}}}\right)$$

Quantifying modeling error



Linear AVO inversion

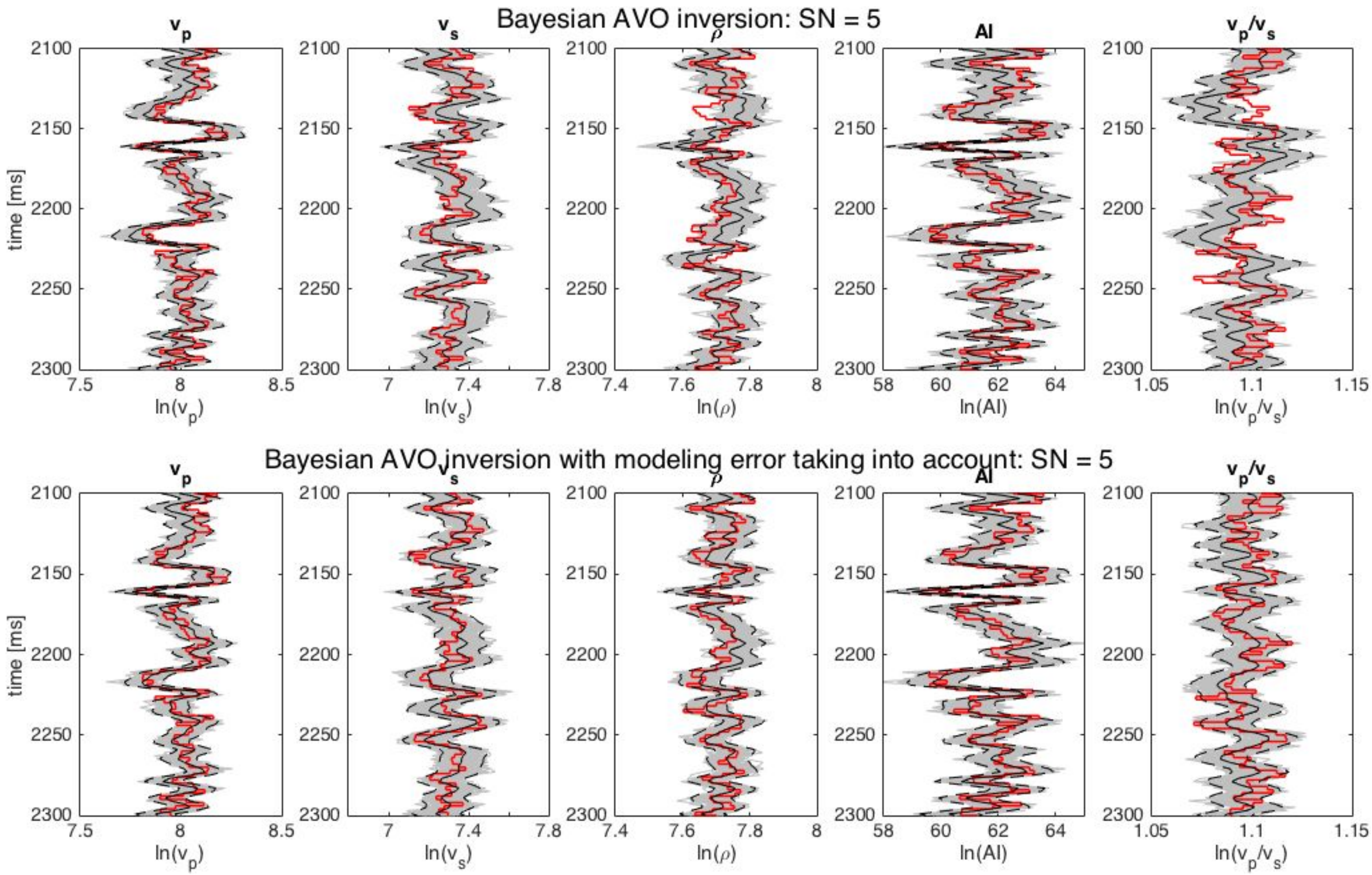
Linear AVO inversion - Setup

- Only possible for Gaussian prior
- Added white noise to reference data
  - SN ratio is constant for all incidence angles
  - S: Standard deviation of signal (zoeppritz)
  - N: Standard deviation of white noise
  - → increasing variance of noise for increasing offset

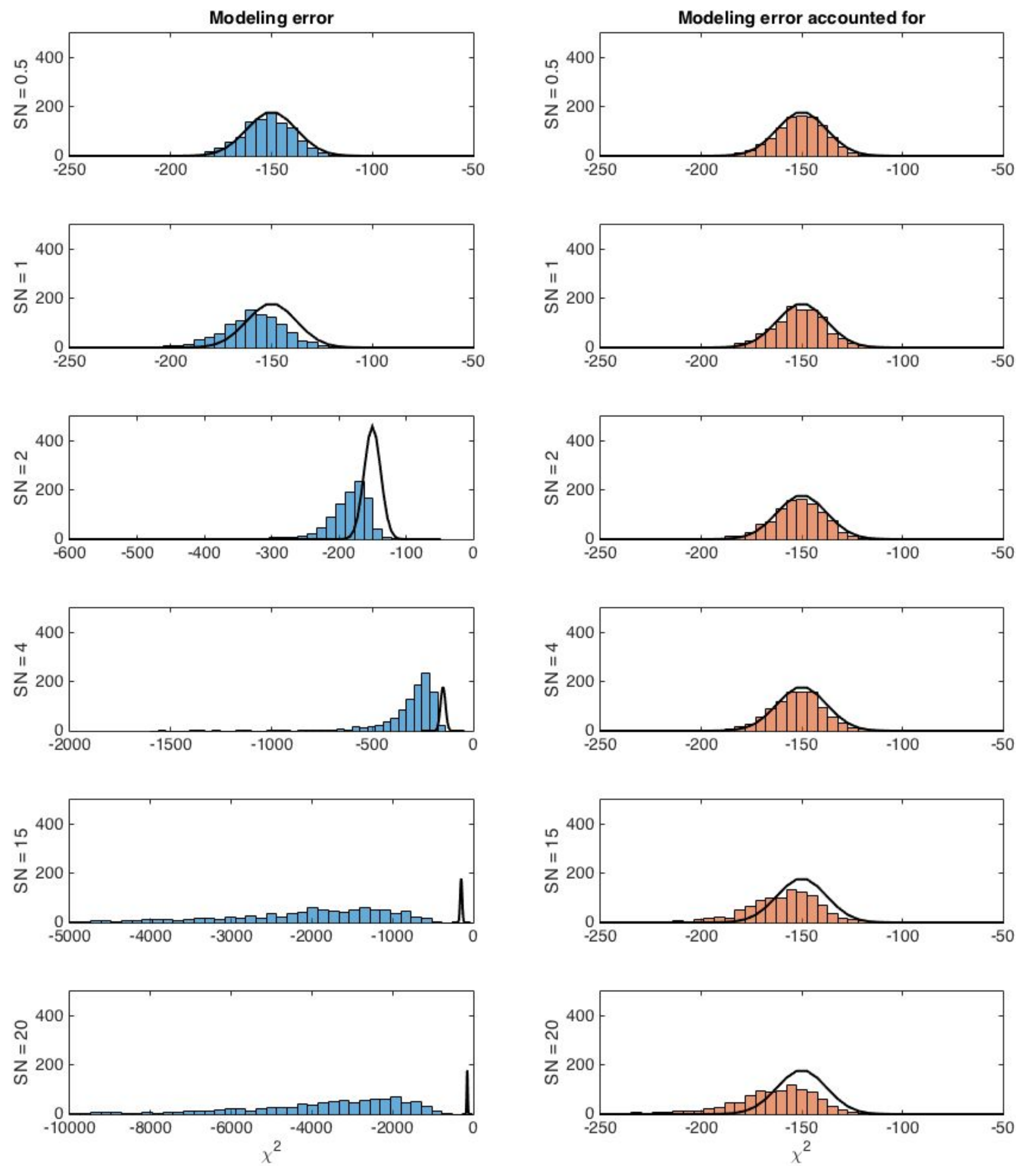
Linear AVO inversion - Posterior

- 'Classic' AVO inversion
  - Noise model: Uncorrelated white noise: Σ_e
- AVO Inversion accounting for modeling error
  - Noise model: Uncorrelated white noise: Σ_e + estimated forward modeling error: C_{Tapp}

Linear AVO inversion



Posterior - Likelihood



In summary

- Choice of prior model determines level of forward modeling error
- Significance of forward modeling error increases with incidence angle
- We are able to quantify the modeling error with a gaussian covariance model
- This enables accounting for modeling errors in linear AVO inversion for Gaussian priors